Analysis of torsional vibration characteristics for multi-rotor and gear-branched systems using finite element method

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CERTIFICATE

This is to certify that the thesis entitled, **"Analysis of torsional vibration characteristics for multi-rotor and gear-branched systems using finite element method"** submitted by Sri **Vishwajeet Kushwaha** in partial fulfillment of the requirements for the award of Bachelor of Technology Degree in Mechanical Engineering at the **National Institute of Technology, Rourkela** (Deemed University) is an authentic work carried out by him under my supervision and guidance.

To the best of my knowledge, the matter embodied in this thesis has not been submitted to any other University/Institute for the award of any Degree or Diploma.

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Abstract

Torsional vibrations occur due torelative oscillatory motions superposed on the mean rotation. Rotating systems at running speed near the natural frequencies are prone to excessive angular deflections and hence large stresses which can cause rotating components failure or premature fatigue failures. The objective of the present work is to study the torsional vibration characteristics of multi-rotor and gear-branched systems suing finite element method. Finite element method (FEM) is a numerical technique based on principle of discretization to find approximate solutions to engineering problems. The information about the natural frequencies for rotating systems can help to avoid system failure by giving the safe operating speed range. In the present work, finite element method has been used to find these natural frequencies for different possible cases of multi-rotor and gear-branched systems. The various mode shapes for several cases are also shown to illustrate the state of the system at natural frequencies. The results obtained have been compared with Holzer's method and transfer matrix method to establish the effectiveness of finite element method for such systems.

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CHAPTER 1

Introduction

Most of the machines used in industries are rotary in nature and hence are subjected to torsional vibrations. These vibrations can damage the machine components. Torsional vibrations are harder to detect and hence are more dangerous. The amplitude of these vibrations can increase significantly if the system speed is close to the natural frequencies of the system, hence it's important to find the natural frequencies for rotating systems for safe operations. Excessive strain at speeds near the natural frequencies causes excessive stress which causes component failures. It can also lead to premature fatigue failures. Rotating systems with rotors or gears are common in engineering, finite element method can be used to find the natural frequencies for straight as well as branched multirotor systems. So the finite element method and characteristics of torsional vibrations have been discussed in detail.

1.1- Finite element method

1.1.1- Fundamental concepts

The following definitions are a prerequisite in orders to understand the finite element analysis associated with any system-

A)Stiffness matrix- It relates a displacement vectors to a force vector. The entries in the stiffness matrix can be identified as the stiffness influence coefficients, which represent a strictly static concept.

Methods to find the stiffness matrix-

1- **Direct Method**- The stiffness influence coefficients are derived by writing the equations of force equilibrium for each node.

2- **Variational** Approach- The stiffness influence coefficients are derived by writing the equations of motions using the kinetic and potential energies for each element.

B)Mass matrix- It relates acceleration vectors to the force vectors. The coefficients of the mass matrix represents the the mass or inertia associated with each element.

C) Steps involved with FEM methodology-

1- Discretize the continuum.

2- Select interpolation functions.

3- Find the element properties.

4- Assemble the element properties to obtain the system equation.

5- Impose the boundary conditions.

6- Solve the system equations.

7- Make additional computations if desired.

D) <u>Graphical summary -</u>

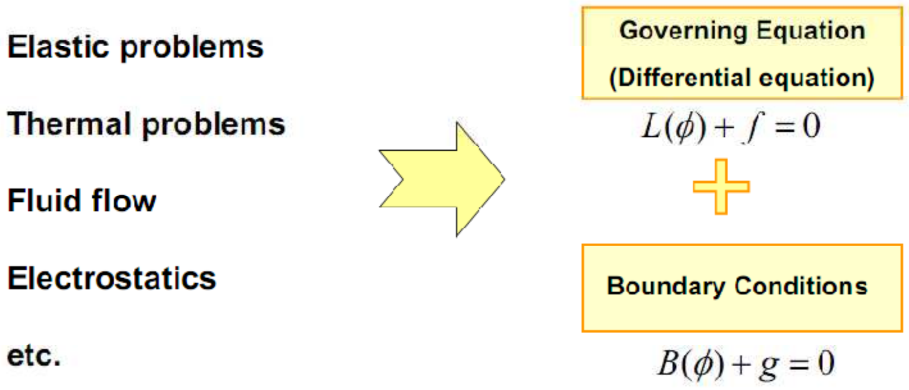


Fig. (1.1)- fundamentals of finite element method

$$\underset{\text{Property}}{[\mathbf{K}]}\underset{\text{Behavior}}{\{\mathbf{u}\}} = \underset{\text{Action}}{\{\mathbf{F}\}} \quad \Rightarrow \quad \{\mathbf{u}\} = [\mathbf{K}]^{-1}\{\mathbf{F}\}$$

1.1.2 – FEM advantages and Disadvantages-

A) Advantages-

Can readily handle very complex geometry:
- The heart and power of the FEM

Can handle a wide variety of engineering problems
- Solid mechanics
- Dynamics
- Heat problems
- Fluids
- Electrostatic problems

Can handle complex restraints
- Indeterminate structures can be solved.

Can handle complex loading
- Nodal load (point loads)
- Element load (pressure, thermal, inertial forces)
- Time or frequency dependent loading

B) Disadvantages-

A general closed-form solution, which would permit one to examine system response to changes in various parameters, is not produced.

The FEM obtains only "approximate" solutions.

The FEM has "inherent" errors.

Mistakes by users can be fatal.

1.2- Introduction to Vibration

1.2.1- Vibration Fundamentals

Vibration is the dynamic behaviour of a system which refers to the system oscillation about equilibrium position.

The physical properties, or characteristics, of a system are referred to as parameters. The analysis can be simplified by replacing the distributed characteristics of a continuous system by discrete ones. This is accomplished by a suitable "lumping" of the continuous system. Hence, mathematical models related to vibration analysis of any system can be divided into two major types:

(1) discrete-parameter systems, or lumped systems.

(2) distributed parameter system, or continuous systems.

Oscillatory systems can be broadly characterized as linear or nonlinear.

Linear systems:

The principle of superposition holds

- Mathematical technique available for their analysis is well developed.

Nonlinear systems:

The principle of superposition doesn't hold

- The technique for the analysis of the nonlinear systems are under development (or less well known) and difficult to apply.

All systems tend to become nonlinear with increasing amplitudes of oscillations.

There are two general classes of vibrations– free and forced.

Free vibrations:

• Free *vibration* takes place when a system oscillates under the action of forces inherent in the system itself due to initial disturbance, and when the externally applied forces are absent.
• The system under free vibration will vibrate at *one or more* of its *natural frequencies*, which are *properties of the dynamical system*, established by its mass and stiffness distribution.

Forced vibrations:

- The vibration that takes place under the excitation of external forces is called forced vibration.
- If excitation is harmonic, the system is forced to vibrate at *excitation frequency* . If the frequency of excitation coincide with one of the natural frequencies of the system, a condition of *resonance* is encountered and dangerously large oscillations may result, which results in failure of major structures, i.e., bridges, buildings, or airplane wings etc.
- Thus calculation of natural frequencies is of major importance in the study of vibrations.
- Because of friction & other resistances vibrating systems are subjected to *damping* to some degree due to dissipation of energy.
- Damping has very *little effect on natural frequency* of the system, and hence the calculations for natural frequencies are generally made on the basis of no damping.
- Damping is of great importance in *limiting the amplitude* of oscillation at resonance.

1.2.2- Torsional Vibration

Torsional vibration is an oscillatory angular motion that causes relative twisting in the rotating members of a system. This oscillatory twisting motion gets appended to the steady rotational motion of the shaft in a rotating or reciprocating machine. Systems in which some driving equipment drives a number of components, thus enabling them to rotate, are often subjected to constant or periodic torsional vibration. This necessitates the analysis of the torsional characteristics of the system components.

Torsional vibrations may result in shafts from following forcings:

- Inertia forces of reciprocating mechanisms (such as pistons in Internal Combustion engines)
- Impulsive loads occurring during a normal machine cycle (e.g. during operations of a punch press)
- Shock loads applied to electrical machineries (such as a generator line fault followed by fault removal and automatic closure)
- Torques related to gear tooth meshing frequencies, turbine blade passing frequencies, etc.

For machines having massive rotors and flexible shafts (where system natural frequencies of torsional vibrations may be close to, or within, the source frequency range during normal operation) torsional vibrations constitute a potential design problem area.

In such cases designers should ensure the accurate prediction of machine torsional frequencies and frequencies of any of the torsional load fluctuations should not coincide with torsional natural frequencies.

Hence, determination of torsional natural frequencies of a dynamic system is very important.

CHAPTER 2

Literature Survey

Literature survey

2.1- Rotating machines

One of the most common machines used are rotating machines. Steam turbines, compressors, generators, pumps etc. are few examples of rotating machines. These rotary systems behave like a torsional oscillator which can exhibit, for various reasons, relative oscillatory motions superposed on the mean rotation. Such is the case of rotor shafts, for which the security of service requires an accurate knowledge of the critical speeds: indeed a rupture can occur when the frequency of a disturbing excitation torque is equal to a natural one of the shaft. Hence, when studying a rotary engine it is first necessary to determine the shaft natural frequencies. Machines running at speeds near their natural frequencies are also prone to premature fatigue failures.

Torsional vibrations are deemed more dangerous than other form of vibrations as these are very hard to detect comparatively. Hence it can lead to abrupt failures and can cause direct malfunctioning and direct breakage of shafts used in the drive trains. Thus it justifies the need for preliminary vibrational analysis during the design stage only so that expensive modifications can be avoided later during operation and manufacturing. Most of the systems involving torsional vibrations can be represented as combination of rigid disks or rotors with fixed mass moment of inertia and shafts with certain torsional stiffness. If the shafts are of small length and diameter, their masses can be neglected while using finite element method which simplifies the problem.

The torsional vibrations of the mechanism shafts have been studied by many researchers The torsional vibrations of linkage mechanisms have been investigated by Meyer zurCapellen [1]. Houben [2] has examined the torsional vibrations of the shafts of machines driven by an electrical motor. The stability of the torsional vibrations of the crankshafts of engines has been investigated by Krumm[3].

2.2- Branched systems

Branched systems can be viewed as a rotor system composed of two or more rotors interacting with each other. Vibration of gear branched system has been studied as an important subject because it is an important part of rotary machines and hence finds application in many industrial applications. They are essentially used to transmit power from one shaft to another as well as for reduction or increase in the speeds. Some examples are ship propulsion systems, turbine systems, drive shaft and differential of the automobiles. As discussed above, shafts rotating at frequencies near their natural frequencies are prone to excessive angular deflections thereby excessive shear stresses which can lead to component failure. So it'sto necessary find the natural frequencies for such rotating systems. FEM has proved to be a powerful tool in solving such problems.Holzer method was used in early days for vibration analysis of direct transmitted or straight systems [4]. the transfer matrix was suitable to use with high speed computers. [5,6]. FEM has also been used for torsional vibrations of straight[7] and a gear-branched system [8, 9]. It's difficult to solve the problems associated with the gear-branched systems as the total number of independent degrees of freedom is less than the total number of rotors.A simple method to eliminate the dependent degrees of freedom has been devised by. Wu and Chen[10] .

CHAPTER 3

Theoretical analysis

The different methods for finding the natural frequencies of any vibrating system are-

1- Dunkerley's method

This method is generally used for transverse vibrations. It's effective in case of low damping and harmonics having frequencies higher than the fundamental frequencies. It gives the lower bound approximation.

2- Reyleigh's method-

It uses the principle that in a conservative system the frequency of vibration has a stationary value in the neighbourhood of a natural mode. It gives the upper bound approximation of the fundamental frequency of the system.

3- Holzer's method-

The natural frequency and mode shapes of a multimass lumped parameter system can be determined by this iteration method as devised by Holzer. It's applicable to forced , free, damped, undamped and semi-definite systems as well.

4- Transfer matrix method-

Holzer's method can be implemented by using the concept of state vector and transfer matrices for continuous as well as discrete systems. It can also be suitably used for branched systems.

5- Finite element method-

This method can be regarded as a general case of Rayleigh-Ritz method. It involves the formulation of the eigenvalue problem by determining the stiffness and mass matrices (using Lagrange's equation or direct method) for each of the elements and their finite assemblage to get the global stiffness and mass matrix respectively.

In the present work, we are mainly concerned with the holzer, transfer matrix and finite element method to determine the fundamental frequencies, so their detailed discussion is as follows:

3.1- Holzer'smethod-

3.1.1- For straight systems

Holzer's method is an iterative method which is used to find the fundamental frequencies and mode shapes of any system with multiple degrees of freedom. In case of straight systems it's basically the proper tabulation of equations of motion governed by newton's 2^{nd}law. It's a very versatile technique and can be used for both branched and unbranched systems with and without damping. We consider here the a typical semi- definite vibrating system as shown in fig below, the holzer's method can be applied to such systems as follows-

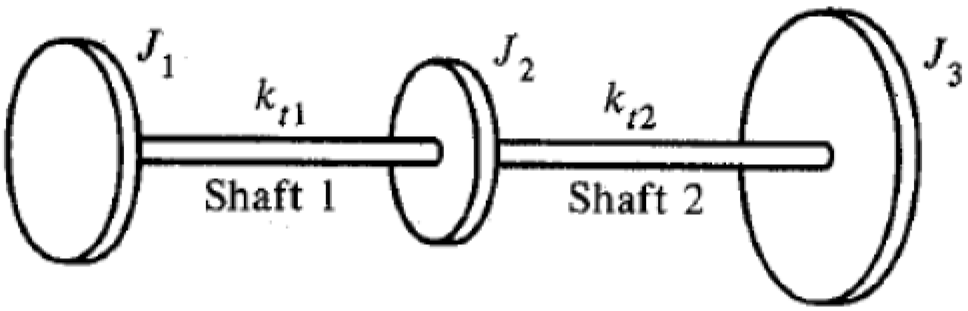


Fig. (3.1)- straight rotor torsional system for Holzer's method

By Newton's second law,

$$\left.\begin{aligned} J_1\ddot{\theta} &= -k_{t1}(\theta_1-\theta_2) \\ J_2\ddot{\theta} &= -k_{t1}(\theta_2-\theta_1) - k_{t2}(\theta_3-\theta_2) \\ J_3\ddot{\theta} &= -k_{t2}(\theta_3-\theta_2) \end{aligned}\right\} \quad (1)$$

The harmonic nature of vibration at principal modes allows us to write,

$$\theta_i = \Theta_i \sin\omega t \quad (2)$$

Putting (2) in (1)

$$-\omega^2 J_1 \ddot{\Theta} = -k_{t1}(\Theta_1 - \Theta_2)$$

$$-\omega^2 J_2 \ddot{\Theta} = -k_{t1}(\Theta_1 - \Theta_2) - k_{t2}(\Theta_3 - \Theta_2) \quad (3)$$

$$-\omega^2 J_3 \ddot{\Theta} = -k_{t2}(\Theta_3 - \Theta_2)$$

Adding the above equations we get,

$$\sum_{i=1}^{3} J_i \, \Theta_i \omega^2 = 0 \quad (4)$$

And for n rotor system,

$$\sum_{i=1}^{n} J_i \, \Theta_i \omega^2 = 0 (5)$$

Generalised for n rotor system to find angular displacement,

$$\Theta_j = \Theta_{j-1} - \frac{\omega^2}{k_{t(j-1)}} \sum_{i=1}^{j-1} J_i \, \Theta_i (6)$$

The above derived equation is used for iteration assuming an intial value of $\omega$and Θ. The values of ω for which (5) is satisfied are the natural frequencies and the values Θ_i give the mode shapes.

3.1.2- For branched systems-

Holzer's method for brached systems is generally defined by transfer matrix method. However if the torsional stiffness of the branch shafts in the equivalent sytem are too large, the rotors attached to the branches can be lumped together and the system reduces to single branched system. [16]

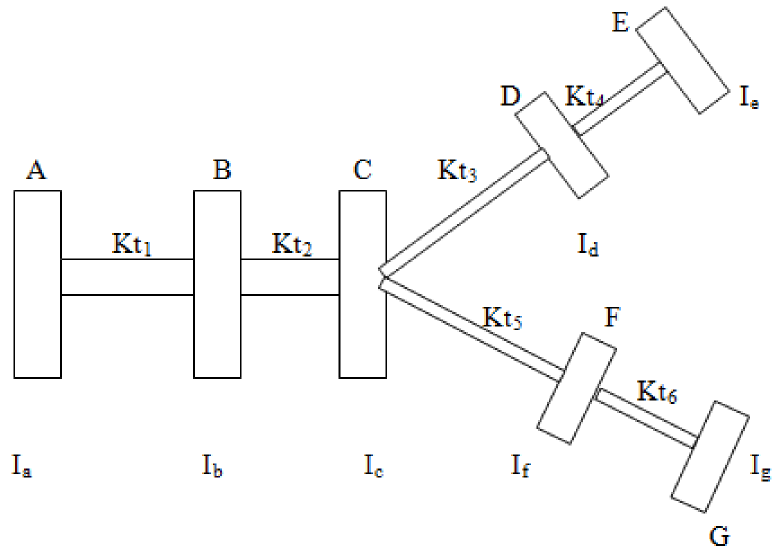


Fig. (3.2) - branched torsional system for Holzer's method

The general form of holzer method can be then applies if we iterate for frequencies such that the deflections associated with the rotors in the two branches are same at the branch point i.e. at the reducing gear arrangement. The new equivalent system with lumped masses can be represented as

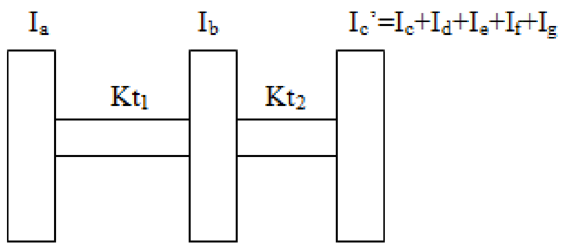


Fig. (3.3)- branched system with lumped masses for Holzer's method

3.2 – Transfer matrix method

consider a branched system as described in [14] is shown in fig. (2) , with three branches A, B and C, denoted by n_a, n_b and n_c. Then the overall transfer matrices are as follows

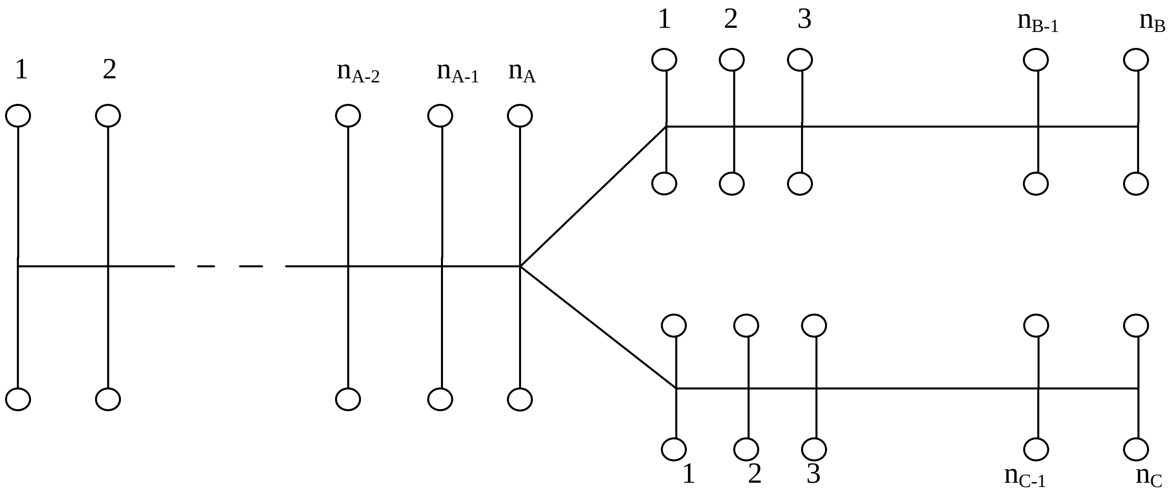


Fig. (3.4)- branched torsional system for transfer matrix method

$$\{S\}_{nA}^{R} = [A]\{S\}_{A0}$$

$$\{S\}_{nB}^{R} = [A]\{S\}_{B0} \tag{7}$$

$$\{S\}_{nC}^{R} = [A]\{S\}_{C0}$$

At the branch point, following conditions are satisfied,

$$\theta_{nA} = \theta_{nB} = \theta_{nC} \qquad (8)$$

$$T_{nA} = T_{nB} = T_{nC}$$

Using the condition of free end 1 of with$\theta_{A0} = 1$, and from (7)

$$\begin{Bmatrix} \theta \\ T \end{Bmatrix} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \tag{9}$$

Hence,

$T_{nA}^{R} = A_{21}$ (10)

$\theta_{nA}^{R} = A_{11}$

Using (10) and (8) in (7)

$\begin{Bmatrix} \theta \\ 0 \end{Bmatrix}_{nB}^{R} = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix} \begin{Bmatrix} A_{11} \\ T \end{Bmatrix}_{B0}$ (11)

Therefore,

$$T_{B0} = -\frac{B_{21}A_{11}}{B_{22}} \qquad (12)$$

Using (10) and (12) in (8),

$$T_{C0} = A_{21} + \frac{B_{21}A_{11}}{B_{22}} \qquad (13)$$

The third equation (7) can now be written as,

$\begin{Bmatrix} \theta \\ 0 \end{Bmatrix}_{nC}^{R} = \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix} \begin{Bmatrix} A_{11} \\ A_{21} + \frac{B_{21}A_{11}}{B_{22}} \end{Bmatrix}$ (14)

The frequency equation is therefore,

$C_{21}A_{11} + C_{22}A_{21+} \frac{C_{22}B_{21}A_{11}}{B_{22}} = 0$

3.3 Finite element method-

If the inertial of the shafts connecting the the rotors is neglected, then the finite element method reduces to representation of the equations of motion for rotor in the form of an eigenvalue problem. The eigenvalues and eigenvectors hence found are the fundamental frequencies and the mode shapes respectively.

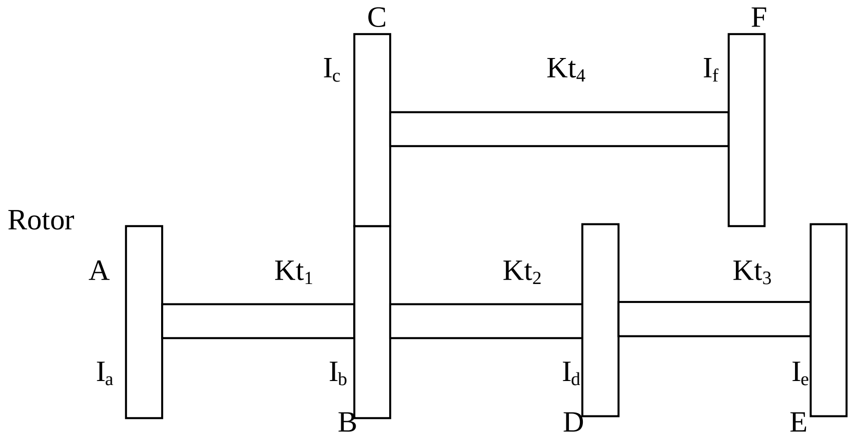


Fig. (3.5)- branched torsional system for finite element method

The method has been illustrated for a special case of branched gear systems. These systems can be reduced to an equivalent form with one to one gear by multiplying all the inertias and stiffnesses of the branches by the squares of their speed ratios.

[we assume that at any instant, $\theta_a, \theta_b, \theta_d, \theta_e, \theta_f'$ are the displacements of various rotors from their equilibrium positions. This means that the twists of the respective shafts are ($\theta_a - \theta_{bc}$), ($\theta_{bc} - \theta_d$),($\theta_d - \theta_e$) and ($\theta_{bc} - \theta_p'$)]

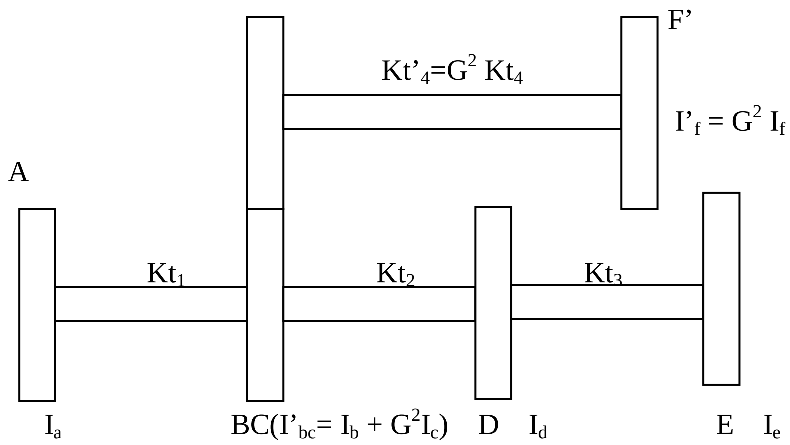


Fig. (3.6)- equivalent branched system for finite element method

From newtons' second law of motion, the equations of motion for the equivalent system can be written as

$$\left.\begin{aligned}
&I_a\ddot{\theta}_a = -Kt_1(\theta_a - \theta_{bc}) \\
&I'_{bc}\ddot{\theta}_{bc} = Kt_1(\theta_a - \theta_{bc}) - Kt_2(\theta_{bc} - \theta_d) - Kt'_4(\theta_{bc} - \theta'_f) \\
&I_d\ddot{\theta}_d = Kt_2(\theta_{bc} - \theta_d) - Kt_3 \qquad 0 \\
&I_e\ddot{\theta}_e = Kt_4(\theta_d - \theta_c) \\
&I'_f\ddot{\theta'}_f = Kt'_4(\theta_{bc} - \theta'_f)
\end{aligned}\right\} \tag{15}$$

Where $I'_{bc} = I_b + G^2 I_c$

$$Kt'_4 = G^2 Kt_4$$

And G=Gear ratio=$\frac{\text{speed of rotor F}}{\text{speed of rotor A}}$

Assuming the motion of the form

$$\left.\begin{aligned}
&\theta_x = A_x cos\omega t \\
&\ddot{\theta}_x = -\omega^2 A_x cos\omega t
\end{aligned}\right\} \tag{16}$$

from (15) and (16) and rearranging, we get

$$\left.\begin{aligned}
&(I_a\omega^2 - Kt_1)A_a + (Kt_1)A_{bc} = 0 \\
&(Kt_1)A_a + (I_{bc}\omega^2 - Kt_1 - Kt_2 - Kt'_4)A_{bc} + (Kt_2)A_d + (Kt'_4)A'_f = 0 \\
&(Kt_2)A_{bc} + (I_d\omega^2 - Kt_2 - Kt_3)A_d + (Kt_3)A_e = 0(17) \\
&(Kt_3)A_d + (I_e\omega^2 - Kt_3)A_e = 0 \\
&(Kt_4)A_{bc} + \left(I'_f\omega^2 - Kt'_4\right)A'_f = 0
\end{aligned}\right\}$$

This is a set of equations in A_a, A_{bc}, A_d, A_e and A'_f

On rearranging the equations (15) further, we get

$$\left.\begin{aligned}
&I_a\ddot{\theta_a} + (Kt_1)\theta_a - (Kt_1)\theta_{bc} = 0\\
&I_{bc}\ddot{\theta_{bc}} - (Kt_1)\theta_a + (Kt_1 + Kt_2 + Kt'_4)\theta_{bc} - (Kt_2)\theta_d - (Kt'_4)\theta'_f = 0\\
&I_d\ddot{\theta_d} - (Kt_2)\theta_{bc} + (Kt_2 + Kt_3)\theta_d - (Kt_3)\theta_e = 0 \qquad (18)\\
&I_e\ddot{\theta_e} - (Kt_3)\theta_d + (Kt_3)\theta_e = 0\\
&I'_f\ddot{\theta_f} - (Kt'_4)\theta_{bc} + Kt'_4\theta'_f = 0
\end{aligned}\right\}$$

From equations (18) , we can determine the determinant form the problem.

The mass and stiffness matrix can be defined from (18) as

The global mass matrix is

$$[M] = \begin{bmatrix} I_a & 0 & 0 & 0 & 0 \\ 0 & I_{bc} & 0 & 0 & 0 \\ 0 & 0 & I_d & 0 & 0 \\ 0 & 0 & 0 & I_e & 0 \\ 0 & 0 & 0 & 0 & I'_f \end{bmatrix}$$

And the global stiffness matrix is

$$[K] = \begin{bmatrix} Kt_1 & -Kt_1 & 0 & 0 & 0 \\ -Kt_1 & (Kt_1 + Kt_2 + Kt'_4) & -Kt_2 & 0 & -Kt'_4 \\ 0 & -Kt_2 & (Kt_2 + Kt_3) & -Kt_3 & 0 \\ 0 & 0 & -Kt_3 & Kt_3 & 0 \\ 0 & -Kt'_4 & 0 & 0 & Kt'_4 \end{bmatrix}$$

$[\mathrm{M}]\{\ddot{\theta}\} + [\mathrm{K}]\{\theta\} = 0$

$-\omega^2[\mathrm{M}] + [\mathrm{K}] = 0$

$\omega^2[\mathrm{M}] - [\mathrm{K}] = 0$

$\lambda[\mathrm{I}] - [\mathrm{M}]^{-1}[\mathrm{K}] = 0$

$\lambda[\mathrm{I}] - [\mathrm{D}] = 0$ where ,$[\mathrm{D}]=[\mathrm{M}]^{-1}[\mathrm{K}]$, $\lambda = \omega^2$

$\mathrm{Det}(\lambda[\mathrm{I}] - [D]) = 0$

CHAPTER 4

Results and discussion

In order to do a comparative study between the results obtained from the Finite Element Method and the Holzer's Method, some typical problems have been solved and their results have been compared. Several lower mode shapes for 6 rotor straight system and branched system have been plotted to illustrate the problems. At the end two C++ programs are appended which have been used to determine the natural frequencies by holzer's method for straight and branched system.

4.1 Straight Systems

Natural frequencies for straight systems have been obtained here by using FEM and holzer's method.

Problem Description-

Moment of inertia for each rotor= **1Kg-m^2 = I**

Length of each shaft = 1m

Diameter for each shaft = 0.212 m

Modulus of rigidity = 5000 N/m^2

Hence, the torsional stiffness =**1 Nm/rad = K**

Case 1- 2 rotor system

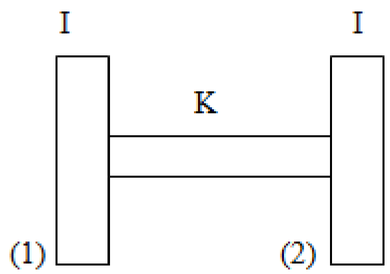
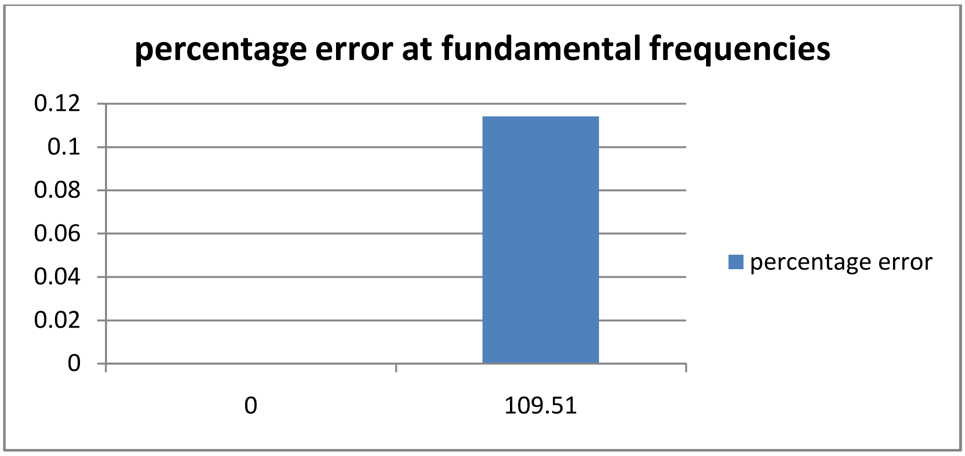


Fig. (4.1)- 2 rotor torsional system

Results-

Table (4.1)- natural frequencies and modes for 2 rotor system

Natural Frequencies	ω1	ω2
Finite element method	0	109.51
Holzer's Method	0	109.385
Absolute difference b/w FEM and Holzer's Method	0	0.125
eigenvectors for the equivalent system	mode1	mode2
rotor1	-0.707	0.707
rotor2	0.707	0.707



Case 2- 3 rotor system

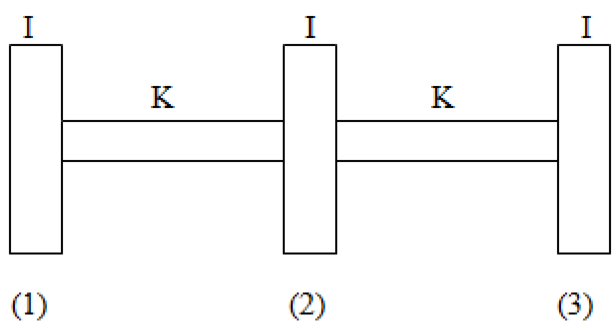
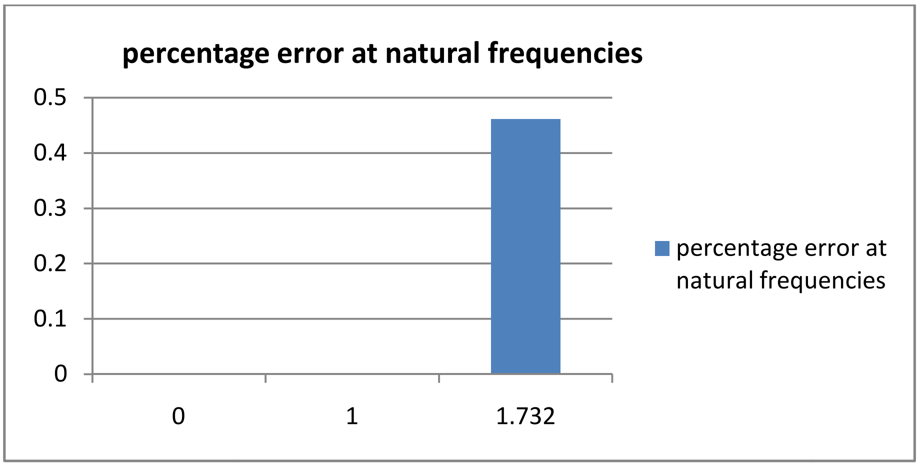


Fig. (4.2)- 3 rotor torsional system.

Results

Table (4.2)- natural frequencies and modes for 3 rotor system

natural frequency(rad/sec)	ω_1	ω_2	ω_3
Finite element method	0	1	1.732
Holzer's method	0	1	1.74
Absolute difference b/w FEM and Holzer's Method	0	0	0.008
Eigenvectors	mode1	mode2	mode3
rotor1	0.577	0.707	-0.408
rotor2	0.577	0	0.816
rotor3	0.577	-0.707	-0.408



Case 3- 4 rotor system

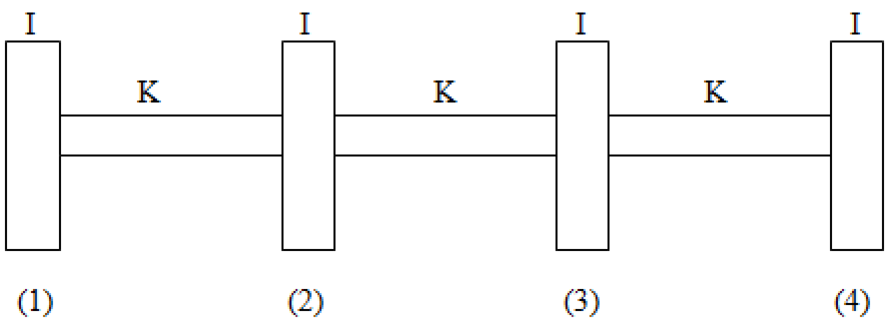
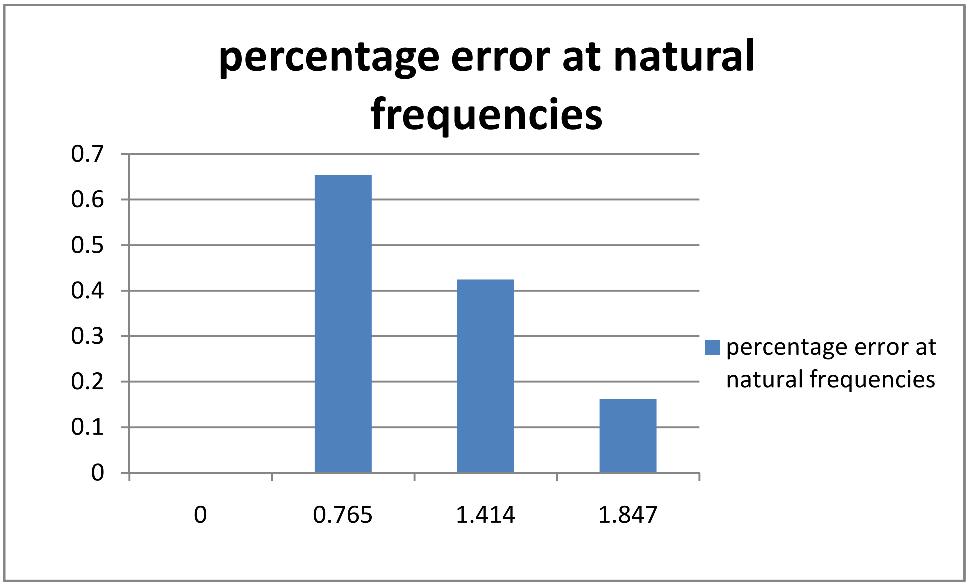


Fig. (4.3)- 4 rotor torsional system

Results-

table (4.3)- natural frequencies and modes for 4 rotor system

natural frequency (rad/sec)	ω_1	ω_2	ω_3	ω_4
Finite element method	0	0.765	1.414	1.847
Holzer's method	0	0.77	1.42	1.85
Absolute difference b/w FEM and Holzer's Method	0	0.005	0.006	0.003
eigenvectors	mode1	mode2	mode3	mode4
rotor1	-0.5	-0.653	0.5	-0.271
rotor2	-0.5	-0.271	-0.5	0.653
rotor3	-0.5	0.271	-0.5	-0.653
rotor4	-0.5	0.653	0.5	0.271



Case 4- 5 rotor system

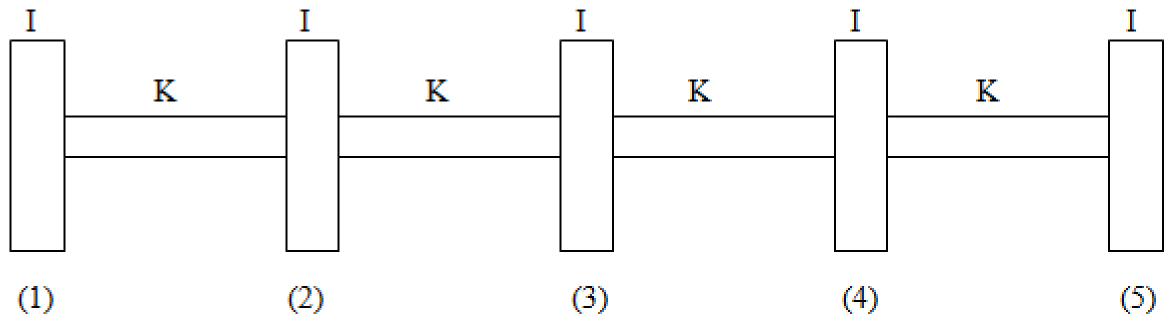
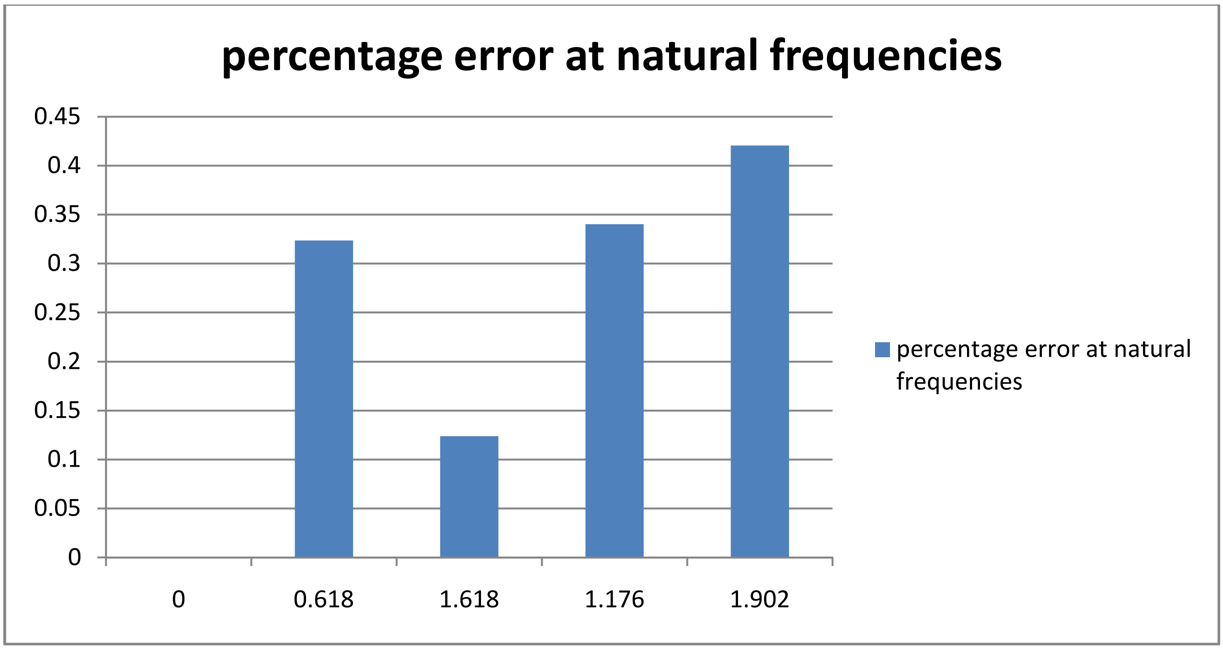


Fig. (4.4)- 5 rotor torsional system

Results

Table (4.4)- natural frequencies and modes for 5 rotor system

natural frequency (rad/sec)	ω_1	ω_2	ω_3	ω_4	ω_5
Finite element method	0	0.618	1.618	1.176	1.902
Holzer's Method	0	0.62	1.62	1.18	1.91
Absolute difference b/w FEM and Holzer's Method	0	0.002	0.002	0.004	0.008
eigenvectors	mode 1	mode 2	mode 3	mode 4	mode 5
rotor1	0.447	0.372	-0.195	0.602	-0.195
rotor2	0.447	0.602	0.512	-0.372	-0.512
rotor3	0.447	0	-0.632	0	0.632
rotor4	0.447	-0.372	-0.195	-0.602	-0.512
rotor5	0.447	-0.602	0.512	-0.372	0.195



Case 5- 6 rotor system

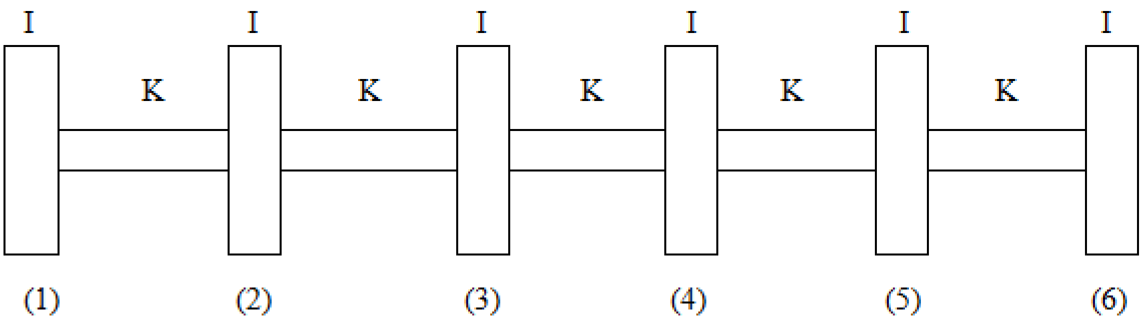
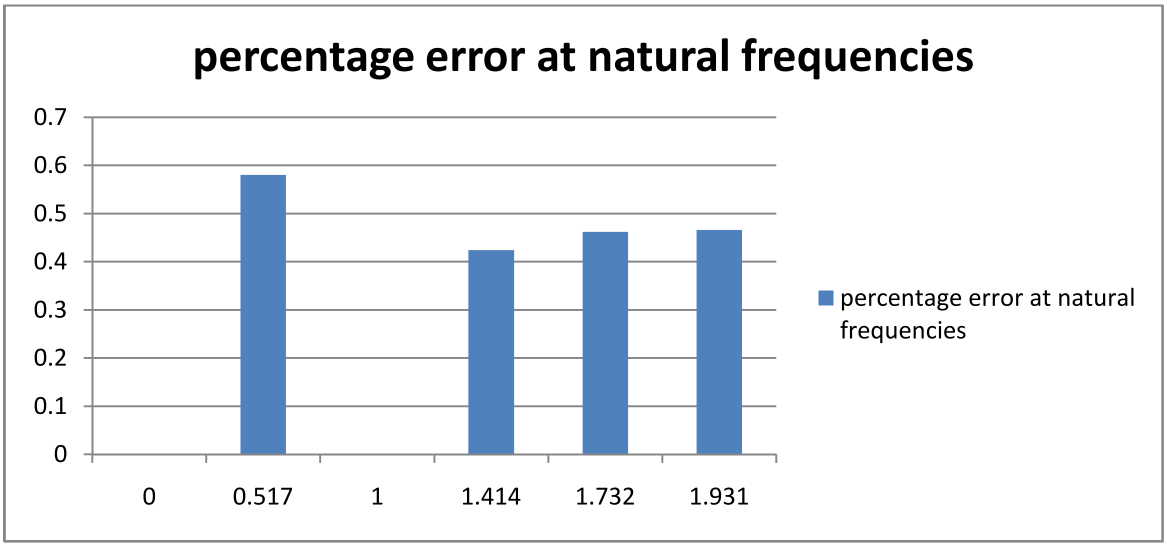


Fig. (4.5)- 6 rotor torsional system

Results-

Table (4.5)- natural frequencies and modes for 6 rotor system

natural frequency (rad/sec)	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6
Finite element method	0	0.517	1	1.414	1.732	1.931
Holzer's Method	0	0.52	1	1.42	1.74	1.94
Absolute difference b/w FEM and Holzer's Method	0	0.003	0	0.006	0.008	0.009
eigenvectors	mode1	mode2	mode3	mode4	mode5	mode6
rotor1	-0.408	-0.558	-0.5	-0.408	0.289	0.149
rotor2	-0.408	-0.408	0	0.408	-0.577	-0.408
rotor3	-0.408	-0.149	0.5	-0.408	0.289	-0.149
rotor4	-0.408	0.149	0.5	0.408	-0.289	-0.558
rotor5	-0.408	0.408	0	-0.408	-0.577	-0.408
rotor6	-0.408	0.558	-0.5	0.408	0.289	0.558



Mode shapes for a six rotor system- the lower 4 mode shapes have been plotted as follows,

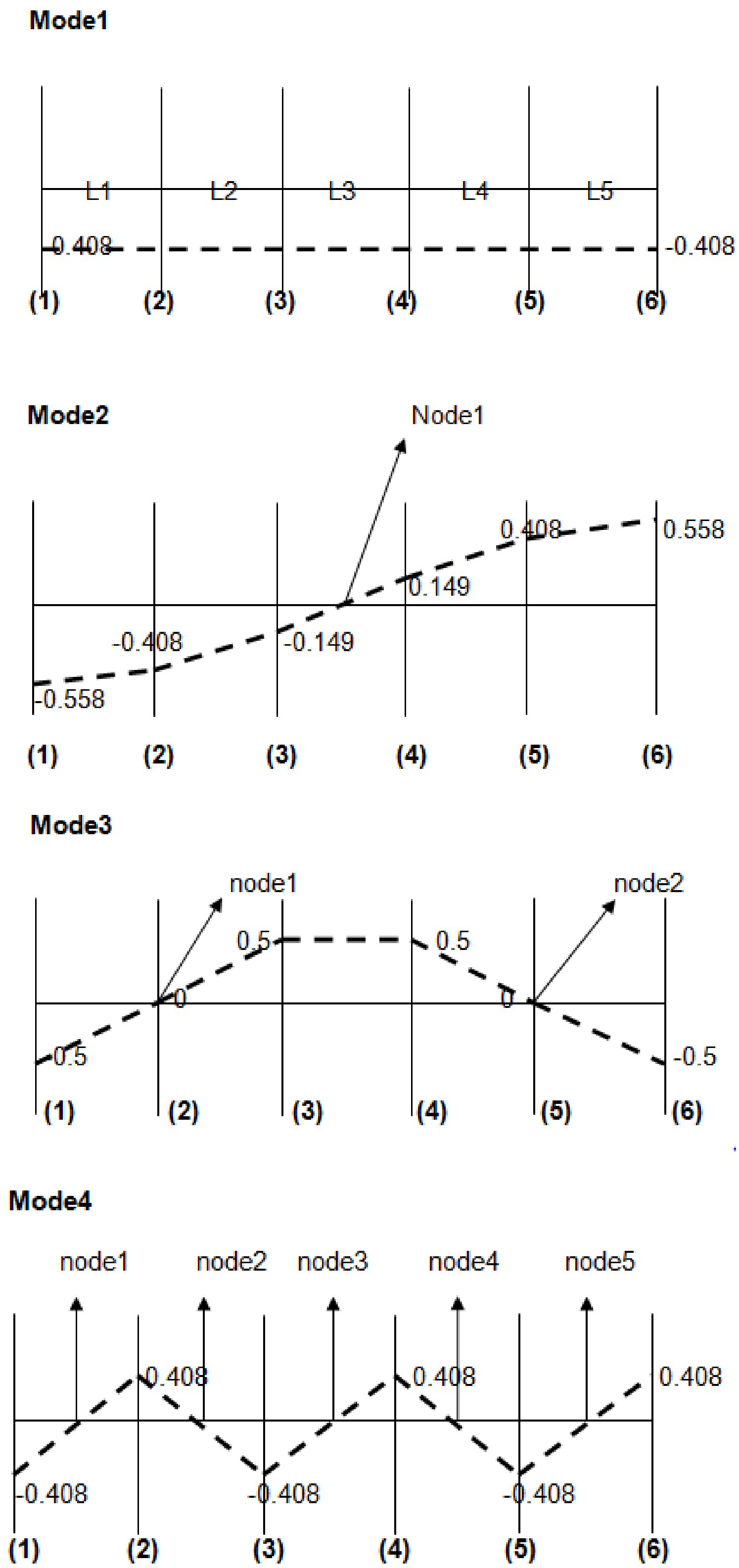


Fig. (4.6) - mode shapes for 6 rotor system

4.2 Branched Systems

4.2.1- Single Branched systems-

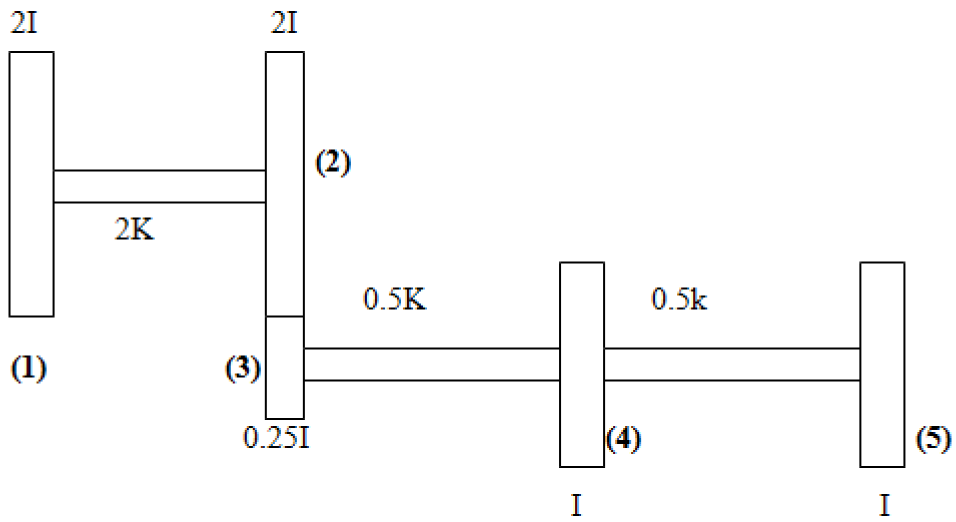


Fig. (4.7) - single branched torsional system

Where I= 13.56 kg-m^2

K= 0.407 x 10^{6}Nm/rad

Gear ratio=D2/D3= 2

The equivalent system for the problem can be represented as,

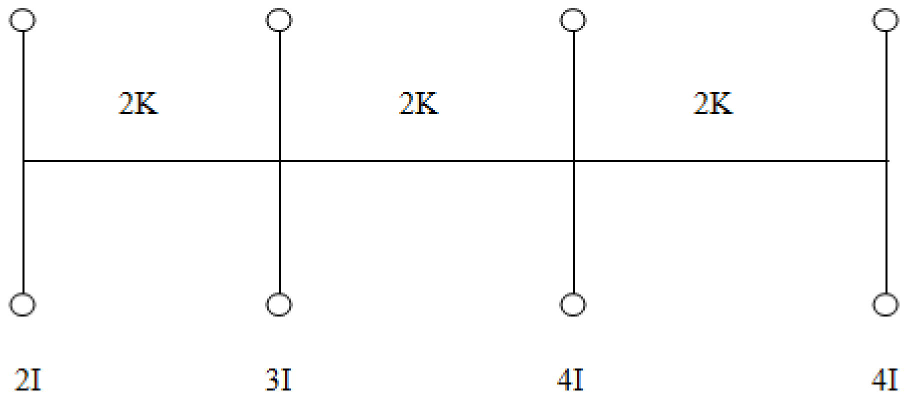


Fig. (4.8)- equivalent system for single branched torsional system

Mass matrix,

$$[M] = \begin{bmatrix} 27.12 & 0 & 0 & 0 \\ 0 & 40.68 & 0 & 0 \\ 0 & 0 & 54.24 & 0 \\ 0 & 0 & 0 & 54.2 \end{bmatrix}$$

Stiffness matrix,

$$[K] = \begin{bmatrix} 812000 & -812000 & 0 & 0 \\ 0 & 1624000 & -812000 & 0 \\ 0 & -812000 & 1624000 & -812000 \\ 0 & 0 & -812000 & 812000 \end{bmatrix}$$

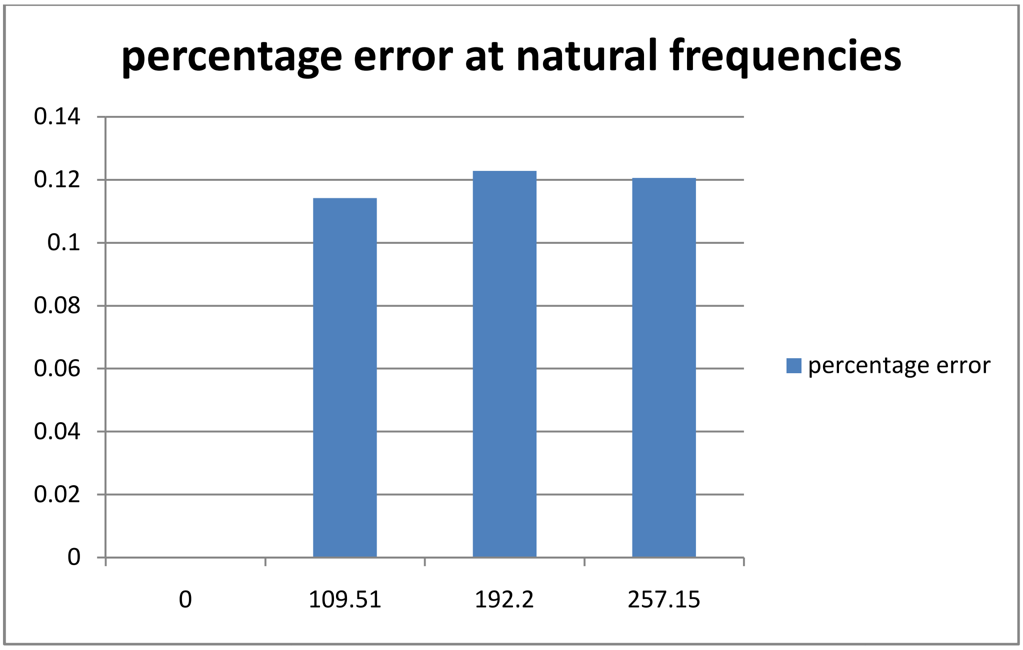
For results we need to solve the eigenvalue problem defined as,

$$\lambda[\mathrm{I}] - [\mathrm{M}]^{-1}[\mathrm{K}] = 0$$

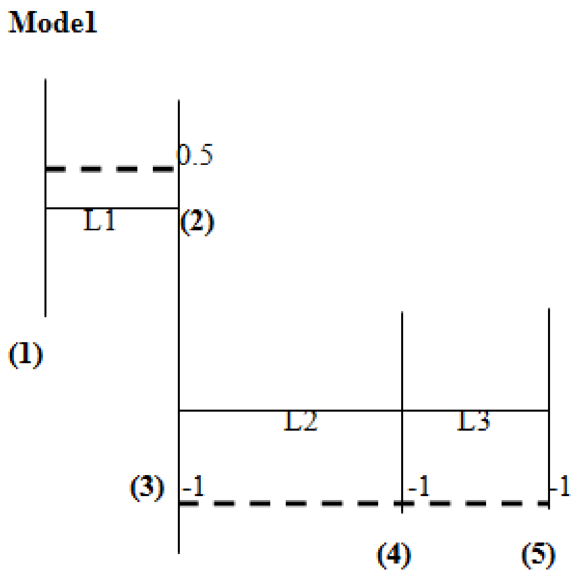
Results

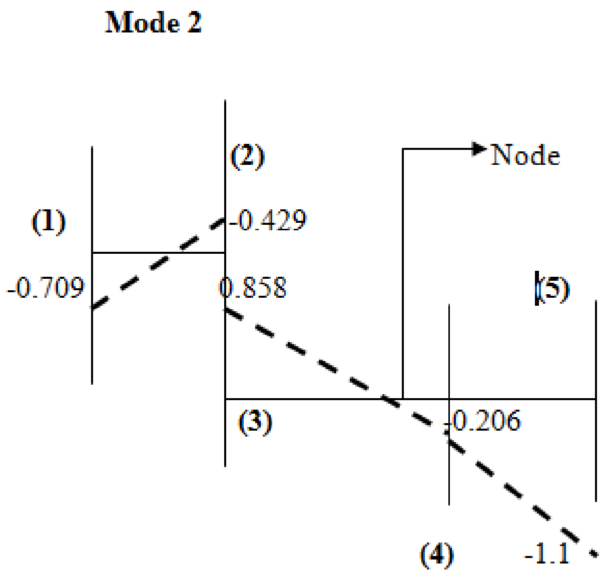
Table (4.6)- natural frequencies and modes for single branched rotor system

Natural Frequencies	ω_1	ω_2	ω_3	ω_4
Finite element method	0	109.51	192.2	257.15
Holzer's Method	0	109.385	191.964	256.84
Absolute difference b/w FEM and Holzer's Method	0	0.125	0.236	0.31
eigenvectors for equivalent system	mode1	mode2	mode3	mode4
rotor1	0.5	-0.709	0.616	0.601
rotor2	0.5	-0.429	-0.127	-0.726
rotor3	0.5	0.103	-0.644	0.322
rotor4	0.5	0.551	0.435	-0.091



Mode shapes-(for the original system, based on eigenvectors obtained for the equivalent System)





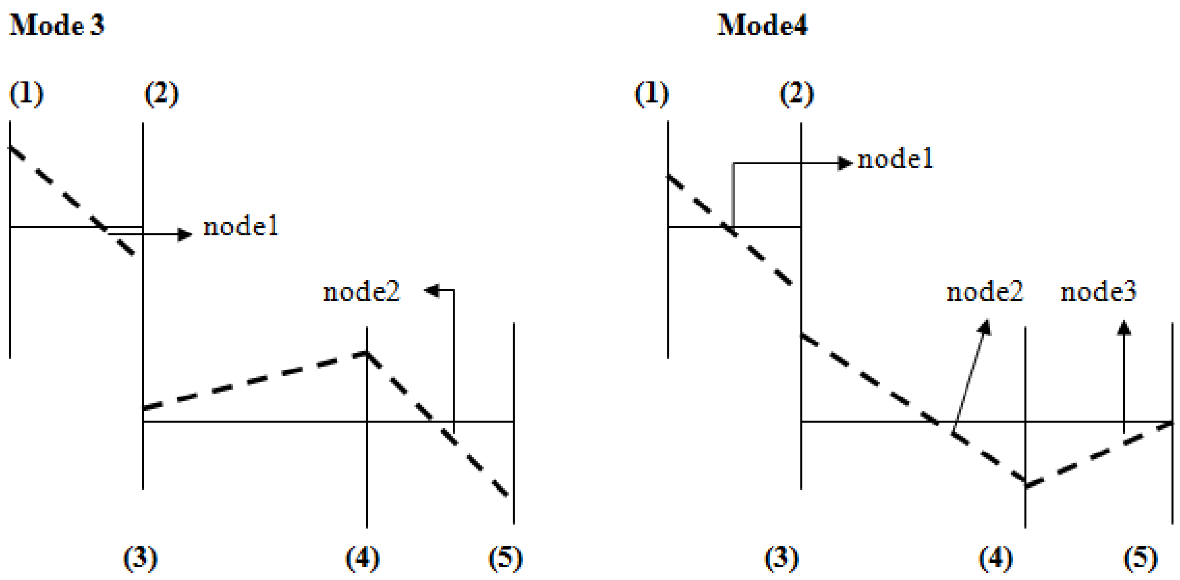


Fig. (4.9) - mode shapes for single branched torsional system

4.2.2- Double Branched systems-

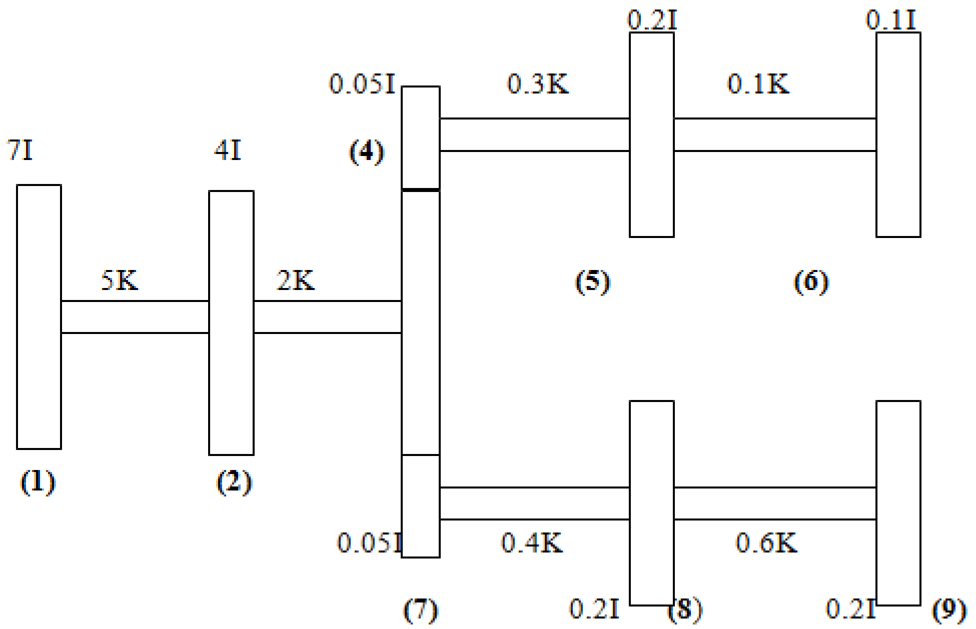


Fig. (4.10) - double branched torsional system

I= 135.5kgm^2

K=4.047 x 10^6 Nm/rad

Gear ratios = G1=G2 =3.16

The equivalent system for the above problem is,

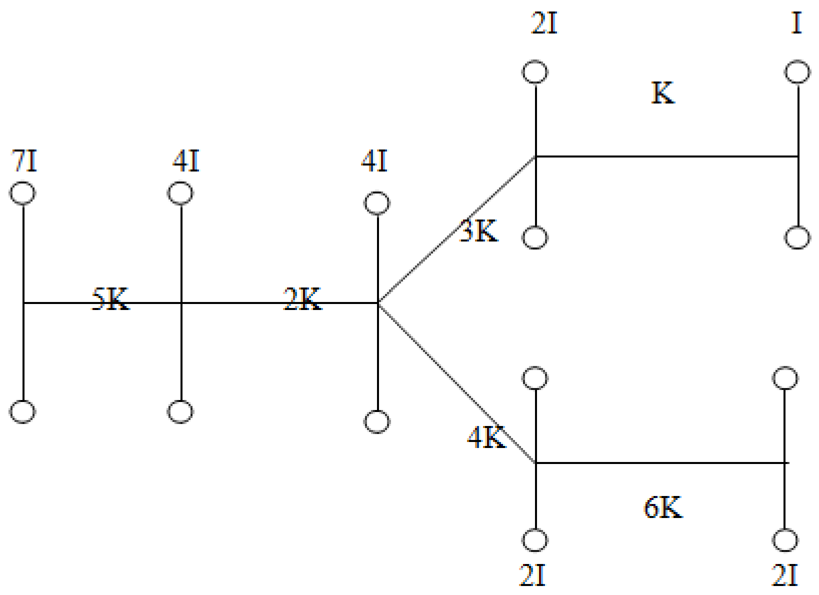
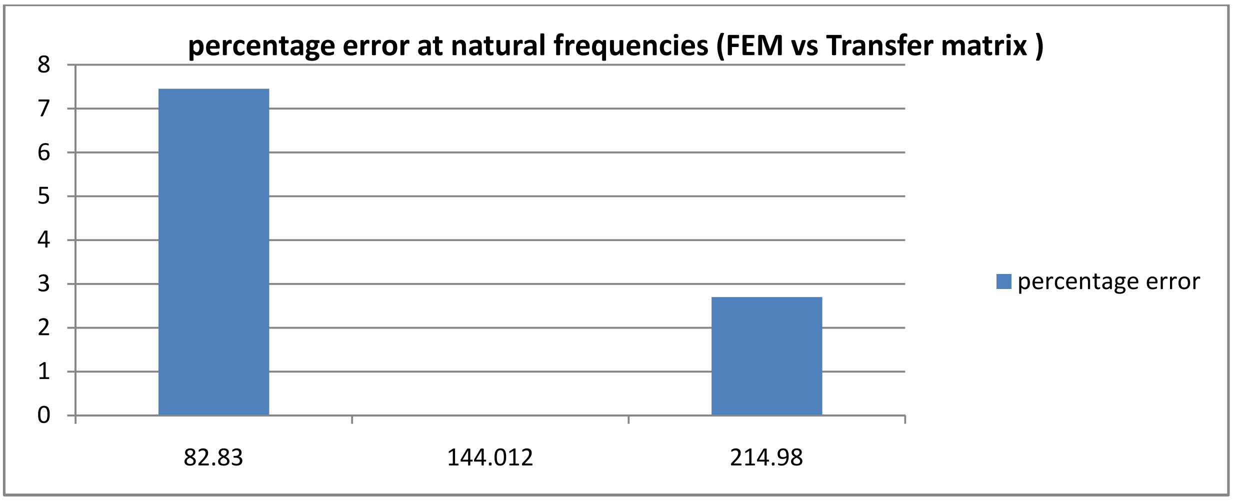


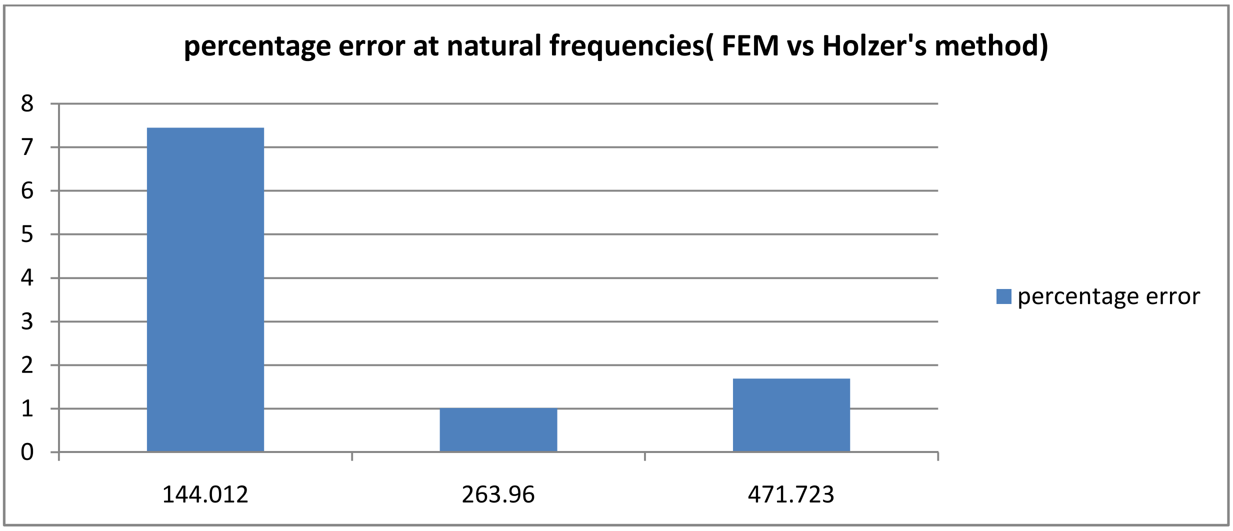
Fig. (4.11)- equivalent system double branched torsional system.

Results

Table (4.7)- natural frequencies and modes for double branched rotor system

Natural Frequencies	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6
Finite element method	82.83	144.012	214.98	263.96	308.127	471.72
Holzer's Method		138		266.63		463.74
Transfer Matrix Method	89	144	220			
Absolute difference b/w FEM and Holzer's Method		6.012		2.67		7.983
Absolute difference b/w FEM andTransfer Matrix Method	6.17	0.012	5.8			
eigenvectors for equivalent system	mode1	mode2	mode3	mode4	mode5	mode6
rotor1	-0.022	-0.557	0.109	-0.326	0.064	0.001
rotor2	-0.001	-0.379	-0.124	0.731	-0.218	-0.014
rotor3	0.054	0.239	-0.327	-0.019	0.456	0.151
rotor4	-0.268	0.32	-0.346	-0.478	-0.734	-0.042
rotor5	-0.863	0.414	0.655	0.362	0.34	0.007
rotor6	0.258	0.314	0.248	0.004	0.016	-0.817
rotor7	0.335	0.34	0.505	0.016	-0.294	0.555


percentage error at natural frequencies (FEM vs Transfer matrix)
8
7
6
5
4
3
2
1
0
82.83
144.012
214.98
percentage error


percentage error at natural frequencies(FEM vs Holzer's method)
8
7
6
5
4
3
2
1
0
144.012
263.96
471.723
percentage error

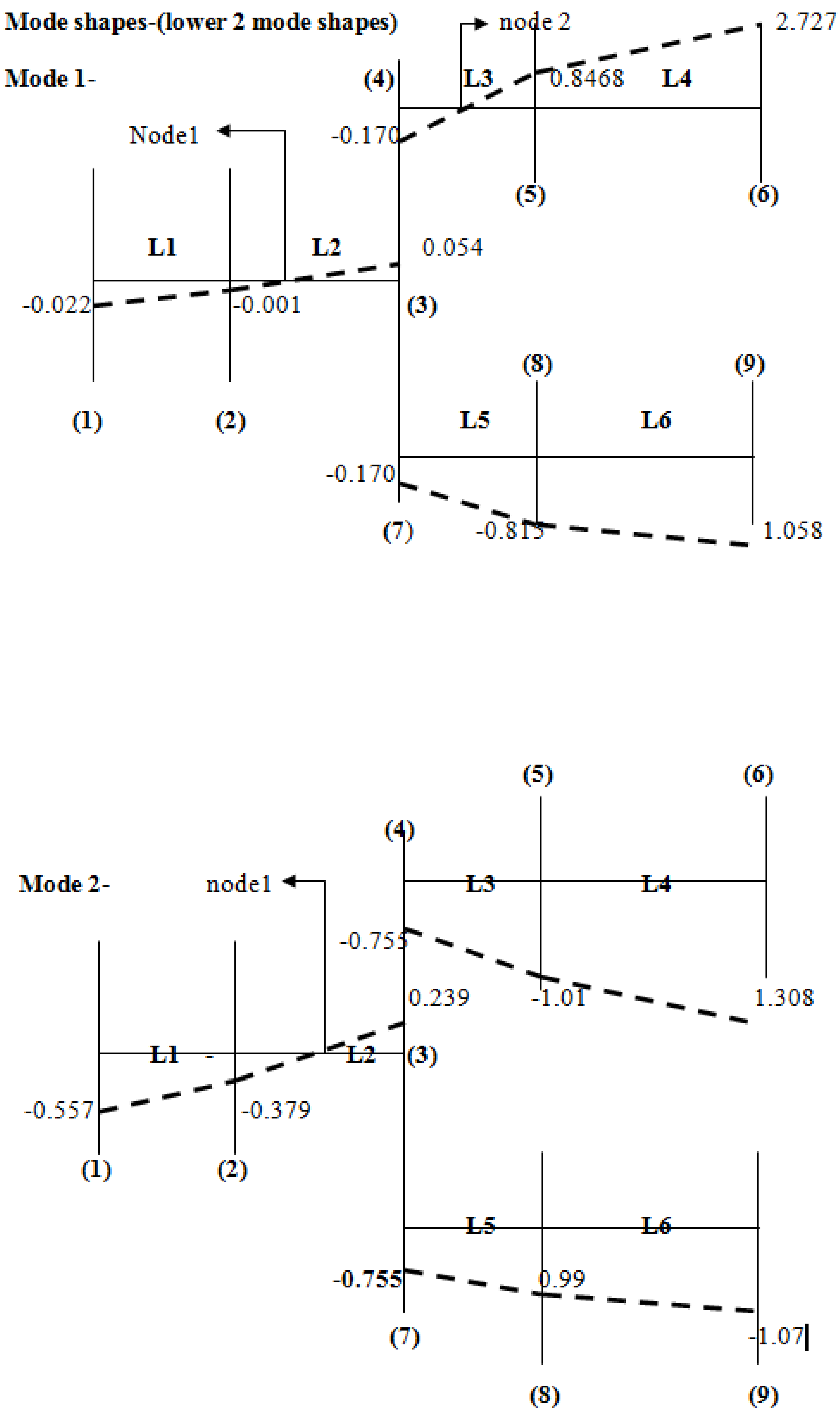


Fig. (4.12)- mode shapes for double branched torsional systems

C++ code to calculate natural frequencies using Holzer's method for straight systems and single branched systems.

The C++ program given below has been developed on the basis of the theory discussed in section 3.1.1. In case of single branch system the program can be used for the equivalent system which acts as a straight or directly transmitted system. The program has been developed by using Dev C++as the compiler and C++ as the programming language.

The results are corrected to three digits after the decimal. All the results for straight and single branched systems have been obtained by using this program.

Code-

```
#include<iostream.h>

#include<conio.h>

int main()

{       float sum,w,t,tprev;  int n;

   cout<<"please enter the number of rotors attached to the shaft(min 2)  ";

   cin>>n;

   float I[n+1],k[n+1],theta[n+1];

   theta[0]=k[0]=I[0]=0;

   //theta[1]=1;

   cout<<"\nenter the inertia for each rotor starting from the free end";

   for(int i=1;i<=n;i++)

   {cout<<"\nfor rotor "<<i<<"  ";

    cin>>I[i];

    };

    cout<<"\nenter the torsional stiffness for the intermediate shafts starting from the free end
";

    for(int i=1;i<=n-1;i++)

   {cout<<"\nfor shaft "<<i<<"  ";

    cin>>k[i];

    };

    w=0.0001;theta[1]=1;

        for(int l=1;l<n;l++)
```

```
{
int flag=1;int slag=1;
while(flag!=0)//t=scalar sum of torque on individual rotors,for equilibrium it should tend
to zero
{ t=0;
for(int i=1;i<=n-1;i++)
{sum=0;
        for(int j=1;j<=i;j++)
        {sum+=I[j]*theta[j]; };
 theta[i+1]=theta[i]-((w*w)/k[i]*sum);
 };
 for(int k=1;k<=n;k++)
 {t+=I[k]*theta[k]*w*w;};
 w=w+0.0001;
 if(slag==1)
 {tprev=t;
 slag=0;
 }
 else
 {if((t/tprev)<0)
 {flag=0;}
  tprev=t;
 }
 };
cout<<"\nthe "<<l<< " non trivial natural frequency is  "<<w-0.0001<<"rad/sec";


}
getch();
return 0;
}
```

C++ code to calculate natural frequencies using Holzer's method for branched systems.

The C++ program given below has been developed on the basis of the theory discussed in section 3.1.2. It has been used to obtain results for the problem solve in section 4.2.2

The data is fed on the basis of the equivalent system and not on the basis of original system.

The branch supporting rotors (7), (8) and (9) in the equivalent system is treated as branch-1 and the other branch supporting rotors(4), (5) and(6) as branch-2 when using the program. The results are obtained upto 3 digits after the decimal. All the results for double branched system using Holzer's method have been obtained by using this program. The compiler used is Dev C++ and the programming language used is C++.

```
#include<iostream.h>

#include<conio.h>

int main()

{//define sum

// program to find the natural frequencies of double branched system at the end

floatsum,w,t,tprev,temp_theta;

int n, b1,b2,count;


cout<<"please enter the number of rotors attached to the main shaft  ";

cin>>n;

cout<<"enter the number of rotors attached to branch1";

cin>>b1;

cout<<"enter the number of rotors attached to branch2";

cin>>b2;

inttotal_rotors;

total_rotors=n+b1+b2;

float I[total_rotors+1],k[total_rotors+1],theta[total_rotors+1];

theta[0]=k[0]=I[0]=0;

   //theta[1]=1;

cout<<"\nenter the inertia for each rotor in the equivalent system starting from the free end of the main shaft";

for(inti=1;i<=total_rotors;i++)
```

```
   {cout<<"\nfor rotor "<<i<<"  ";
cin>>I[i];
    };
cout<<"\nenter the torsional stiffness for the intermediate shafts in equilvalent system starting from the free end of the main shaft";
for(inti=1;i<=total_rotors-1;i++)
   {cout<<"\nfor shaft "<<i<<"  ";
cin>>k[i];
    };
    w=0.0001;


for(int l=1;l<total_rotors;l++)
    {
int flag=1;int slag=1;
while(flag!=0)//t=scalar sum of torque on individual rotors,for equilibrium it should tend to zero
{ t=0;theta[total_rotors]=1;theta[n+b1]=1;sum=0;
for(inti=total_rotors-1;i>n+b1;i--)
    {
sum+=I[i+1]*theta[i+1];
theta[i]=theta[i+1]-((w*w)/k[i]*sum);


  }
sum+=I[n+b1+1]*theta[n+b1+1];
theta[n]=theta[n+b1+1]-((w*w)/k[n+b1]*sum);
temp_theta=theta[n];
sum=0;
for(int j=n+b1;j>n;j--)
     {       sum+=I[j]*theta[j];
theta[j-1]=theta[j]-((w*w)/k[j-1]*sum);
```

```
	}
theta[n]=theta[n]/temp_theta;
sum=0;
for(int m=n;m>1;m--)
	{sum+=I[m]*theta[m];
theta[m-1]=theta[m]-((w*w)/k[m-1]*sum);
	t+=I[m]*theta[m]*w*w;
	}
	w=w+0.0001;
if(slag==1)
	{tprev=t;
slag=0;
	}
else
	{if((t/tprev)<0)
	{flag=0;}
tprev=t;
	}
 };
cout<<"\nthe "<<l<< " non trivial natural frequency is  "<<w-0.0001<<"rad/sec";


      }
getch();
return 0;
}
```

4.3- Discussion

Natural frequencies for different cases have been calculated and the corresponding mode shapes have been plotted for several cases. The differences between the results as obtained by Holzer's and finite element method for each case are shown as percentage error using a bar graph.

For straight systems- for straight system, the percentage error is less than 1 % in all the cases. Thus we can see that the results obtained from these two methods are quite in good agreement with each other. So either of these two methods can be used for straight systems.

For Branched systems-

a) Single branched systems- in case of single branched systems, the system readily reduces to a straight equivalent system, hence, the same procedure as applied for straight systems is applied here, so the percentage error is still very small as in previous cases. However, we can choose between FEM and Holzer's method depending on the time required to solve the problem using Holzer's method as it is iterative in nature and hence can be time consuming.
b) Double branched systems- in case of double branched systems the equivalent system still remains branched at a point but the angular displacement transmitted can be considered to be same for all the branches. Holzer's method in this case is more difficult as the trial frequency for different branches should be such that the displacement gets equal for both the branches at the branching point. Moreover the lumping of masses [16] of the two branches in a single rotor creates room for sufficient error margin. As we can see from table (4.7) the difference between results obtained from these two methods is considerable and varies from 1 to 7%. So FEM is a better option between these two, for branched systems. In case of transfer matrix

method, no such assumptions are made as those used for Holzer's method and hence results are more accurate, as it can be seen the percentage error is less that 1% for one of the natural frequencies.

4.4- Conclusion

The study of torsional vibrations is very important as it can cause component failures or excessive deflections which can lead to premature fatigue failures if the machine is vibrating at speeds near the natural frequencies of the vibrating system. Different methods have been discussed to find these natural frequencies. FEM is the most effective way as it provides a global mass and stiffness matrix which can be derived by discretizing the system and assembling property matrices of the elements. Holzer's method has specifically been used to have a comparative study between the two methods. It being an iteration method based on hit and trial takes long time for complex system with branches. Several cases of single and branched system have been studied and the results obtained by FEM are in good agreement with those obtained by the Holzer's method. Different lower mode shapes have been plotted for some cases for a better understanding of the results obtained. The percentage error in results with respect to FEM has been shown as the bar graph for all the cases. Thus we can see that FEM is a very effective method to calculate the natural frequencies and the corresponding mode shapes. Moreover, the ease of its modelling, less time consumption and its capability to solve intricate problems makes it a very effective tool for different engineering purposes.

4.5 Scope for future work

With the advancement in technology more complex rotating systems are coming into picture with more rotors and branches. The cost involved in repairing and replacements due to failures caused by torsional vibrations can be avoided if the torsional vibration characteristics are considered during the design stages. In the present work the shafts inertias have been neglected due to their low masses but there are systems with large and heavy shafts where the shaft inertia cannot be neglected and has to be taken account which will change the vibration characteristics of the system. Finite element method can be used for such systems also. So there is a lot of work possible in this area with the help of finite element method.

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